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THE STRATIFIED FLOW OF GAS AND NON-NEWTONIAN LIQUID IN HORIZONTAL PIPES

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Abstract—Predictions of pressure drop and holdup are presented for the stratified flow of gas and non-Newtonian liquid obeying the Ostwald–de Waele power law model. The model of Taitel & Dukler (1976) for gas/Newtonian liquid flow is extended to liquids possessing either shear-thinning or shear-thickening laminar flow behaviour and computed results are given for flow behaviour indices in the range $0.1 \le n \le 2$. In particular, conditions are defined for drag reduction of the liquid flow by the presence of the gas. It is concluded that drag reduction occurs over the largest ranges of liquid and gas flow rates at the lowest n values, provided that liquid flow remains laminar, but that maximum drag reduction may be expected for shear-thickening liquids with n values of 2 or greater. Ratios of the liquid flow rate in the presence of gas to that for liquid flow alone under a constant pressure gradient are also presented. These ratios frequently exceed unity and are greatest for highly shear-thinning liquids.

Although the Taitel & Dukler approach is consistent with experiments on gas/Newtonian liquid flow, and, in addition, appears to be valid for immiscible Newtonian liquid–liquid systems, provided that the viscosity ratio of the two phases is at least five, experiments are required to confirm its applicability for gas/non-Newtonian systems.

INTRODUCTION

It is well established that the pressure gradient of a high viscosity liquid flowing in a horizontal pipeline may be reduced by the presence of a much lower viscosity liquid, which is immiscible with the viscous liquid. An example of this phenomenon is the process patented by Clark & Shapiro (1949) in which water is injected into a crude oil pipeline; pressure gradients were reduced by an order of magnitude. Mathematical analyses of the flow were undertaken by Russell & Charles (1959) and Charles & Redberger (1962) and substantial pressure gradient reduction factors were predicted.

Several studies have shown that a similar drag reduction effect occurs for stratified flow of gas and Newtonian liquid, although the authors have not always observed or stated the effect in their publications. Agrawal *et al.* (1973) conducted experiments on gas–oil stratified flow in a 25.8-mm-i.d. pipeline and their data show that percentage drag reductions up to ~50% were obtained at a superficial oil velocity of 0.014 m/s. Drag reduction is not immediately evident from their figures since the pressure gradient is not presented for oil flow alone at the same volumetric flow rate as in two-phase flow. At a higher oil velocity of 0.0616 m/s, a 30% pressure gradient reduction may also be observed. Asaturyan *et al.* (1973) have undertaken a theoretical analysis of laminar-liquid, turbulent-gas stratified flow in order to predict the ratio of liquid flow rate in the presence of gas to that existing for liquid flow alone, under a constant pressure gradient in both cases. Ratios considerably in excess of unity were tabulated for wide ranges of liquid holdup and varying shear stress contributions from interfacial shear between gas and liquid.

In an attempt to justify the general form of the widely-used Lockhart–Martinelli (1949) correlations for holdup and two-phase pressure gradient, Taitel & Dukler (1976) have carried out a simplified analysis of stratified flow of gas and Newtonian liquid. Their predictions agree well with the experimental data of Agrawal *et al.* (1973) and also Russell *et al.* (1974), who used both air–water and air–glycerol/water mixtures flowing in tubes of 25.4-, 38.1- and 50.8-mm-i.d. However, the drag reduction effect was obscurred through the use by Taitel & Dukler of the Lockhart–Martinelli parameter ϕ_G^2, the ratio of the two-phase pressure gradient to the pressure gradient for gas flow alone at the same volumetric flow rate, rather than the similarly-defined parameter ϕ_L^2, based on the liquid flow. Values of ϕ_L^2 less than unity indicate drag reduction, whereas values of ϕ_G^2 greater than unity may or may not be associated with the phenomenon. It is of interest to note that the correlations of Lockhart & Martinelli always give ϕ_L^2 values greater than unity, suggesting that drag reduction does not occur in any flow regime for gas and Newtonian liquid.

The general validity of the Taitel & Dukler model is further substantiated if the model is applied to liquid–liquid systems, provided that there exists a substantial viscosity difference between the phases. This criterion is necessary in order that the more viscous liquid can be considered to flow with a "free" surface at the liquid–liquid interface, while the less viscous liquid is visualised as flowing in a closed duct. Using three different Newtonian liquid–liquid systems with viscosity ratios varying from 5.33 to 20.2 flowing in conduits of both circular and rectangular cross-section, Charles & Lilleleht (1966) correlated laminar-turbulent pressure drop data using parameters similar to those of Lockhart & Martinelli (1949). The parameter ϕ_M^2 was defined as the ratio of the two-phase pressure gradient to the pressure gradient for the flow of the more viscous liquid at the same volumetric flow rate of more viscous liquid, while X^2 was defined as the ratio of the pressure gradient for the flow of the more viscous phase alone to that for the flow of the less viscous phase alone. Table 1 contains coordinates of the mean curves drawn through their data and may be compared with predictions for laminar Newtonian liquid–turbulent gas stratified flow from the Taitel & Dukler model. Excellent agreement is evident over the range $0.05 \le X^2 \le 10$, but at values of $X^2 > 10$ drag reduction is not predicted by the Charles & Lilleleht correlation although one of their ϕ_M^2 values fell below unity at $X^2 = 19$. Charles & Lilleleht comment that the general trend of their data would appear to indicate that, at values of X^2 in excess of 30, ϕ_M^2 could possibly become less than unity. This has yet to be shown experimentally.

Table 1. ϕ^2 Values for laminar liquid/turbulent gas or liquid stratified flow for Newtonian systems

X^2	0.01	0.05	0.1	0.2	0.5	1	2	5	10	20	100
ϕ_M^2 Charles–Lilleleht	—	24.0	12.0	7.05	4.0	2.72	1.92	1.40	1.18	1.07	—
ϕ_L^2 from Taitel–Dukler model	112	25.7	14.4	8.25	4.4	2.78	2.00	1.37	1.11	0.94	0.765
ϕ_L^2 Lockhart–Martinelli	222	73	49	29	16.5	12.1	9.8	6.5	5.2	4.0	2.53

Table 1 includes also values of ϕ_L^2 given by the Lockhart–Martinelli laminar liquid-turbulent gas correlation. Two-phase pressure drops are in general over-predicted by a factor of 2 at low X^2 and as much as 4 at $X^2 = 20$, and it may be concluded that the Lockhart–Martinelli line is inadequate for the stratified flow regime.

The purpose of the present work is to extend the Taitel and Dukler analysis to gas/non-Newtonian liquid stratified flow, where the liquid is in laminar flow and may be described by the Ostwald–de Waele power law model. Heywood & Richardson (1978) have shown recently that large reductions in pressure gradient are possible by the injection of air into a horizontal pipe carrying highly shear-thinning suspensions of flocculated koalin or anthracite, provided the suspension is initially in laminar flow prior to air injection. Owing to a limitation on the lowest superficial suspension velocity which could be measured accurately (0.25 m/s), only the plug (elongated bubble) and slug flow regimes were investigated; stratified flow could be obtained only at order of magnitude lower suspension velocities. The extended analysis presented here is not restricted to shear-thinning, or pseudo-plastic, liquids, but includes also predictions of drag reduction and corresponding liquid holdup for shear-thickening materials. Unfortunately, there is currently no published experimental data for gas/non-Newtonian stratified flow known to the authors. It is hoped that this will be rectified in the future so that the present predictions may be adequately tested.

MODEL DEVELOPMENT

Taitel & Dukler (1976) began their stratified flow analysis by writing a momentum balance for each phase:

$$-A_L\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{tp} - \tau_{WL}S_L + \tau_i S_i = 0 \tag{1}$$

$$-A_G\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{tp} - \tau_{WG}S_G - \tau_i S_i = 0 \tag{2}$$

where A is the flow area for each fluid; τ_W is the shear stress acting at the pipe wall; and S is the surface area per unit length of pipe over which the shear stress acts for each fluid. τ_i is the interfacial stress acting on S_i which is in the positive x direction for the liquid and in the negative direction for the gas since the average gas velocity is generally greater than the average liquid velocity. By making the assumption that the two-phase pressure gradient, $(\mathrm{d}P/\mathrm{d}x)_{tp}$, is the same in both phases, [1] and [2] were combined and a general dimensionless expression given:

$$-\frac{f_L}{f_G}\cdot\frac{(\rho_L U_{LS}^2/2)}{(\rho_G U_{GS}^2/2)}\cdot\frac{\bar{S}_L}{\bar{A}_L}\cdot\bar{U}_L^{\,2} + \left[\frac{\bar{S}_G}{\bar{A}_G} + \frac{f_i}{f_G}\left(\frac{\bar{S}_i}{\bar{A}_L} + \frac{\bar{S}_i}{\bar{A}_G}\right)\right]\bar{U}_G^{\,2} = 0. \tag{3}$$

The reference variables for the dimensionless quantities were chosen as pipe diameter D for length, D^2 for area and the gas and liquid superficial velocities, U_{GS} and U_{LS}. The dimensionless variables in [3] are denoted by an overbar. Thus, $\bar{D}_L = D_L/D$, $\bar{S}_L = S_L/D$, $\bar{A}_L = A_L/D^2$, $\bar{U}_L = U_L/U_{LS}$, etc. The Fanning friction factors are defined by

$$\tau_{WL} = f_L\frac{\rho_L U_L^{\,2}}{2}; \qquad \tau_{WG} = f_G\frac{\rho_G U_G^{\,2}}{2} \tag{4}$$

and

$$\tau_i = f_i\frac{\rho_G(U_G - U_L)^2}{2} \simeq f_i\frac{\rho_G U_G^{\,2}}{2} \quad \text{if} \quad U_G \gg U_L \tag{5}$$

where ρ and U are the density and average velocity of each phase. Densities of both gas and liquid are assumed constant throughout.

Newtonian expressions for either laminar or turbulent flow were used by Taitel & Dukler for f_L and f_G and the simplifying assumption of $f_i = f_G$ was made. The significance of this model may be extended to non-Newtonian materials by assuming that the liquid phase rheological behaviour obeys the commonly-encountered Ostwald–de Waele power law model:

$$\tau = K\dot{\gamma}^n \tag{6}$$

in which K is the consistency index, $\dot{\gamma}$ the shear rate and n the flow behaviour index. The appropriate Reynolds number for pipe flow is

$$\mathrm{Re} = \frac{D_L^{\,n}\cdot U_L^{\,2-n}\cdot\rho_L}{\gamma} \tag{7}$$

where

$$\gamma = K\cdot 8^{n-1}\left(\frac{1+3n}{4n}\right)^n \tag{8}$$

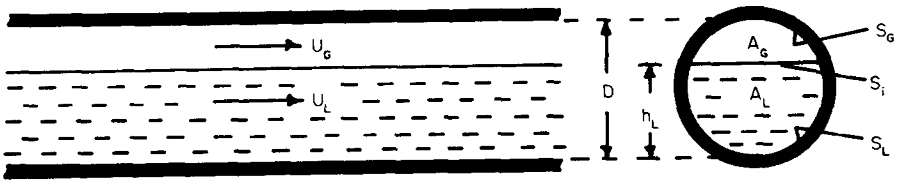

Figure 1. Stratified flow parameters.

and D_L is the hydraulic diameter for liquid flow. The liquid is visualised by Taitel & Dukler (1976) and Agrawal *et al.* (1973) as if it has a "free" surface, as in open channel flow, while the gas is considered to be flowing in a closed duct. Thus, the appropriate definitions of the two hydraulic diameters are (figure 1):

$$D_L = 4\frac{A_L}{S_L} \quad \text{and} \quad D_G = \frac{4A_G}{(S_i + S_G)}. \qquad [9]$$

The liquid phase friction factor for laminar flow conditions is assumed to be given by

$$f_L = \frac{16}{\mathrm{Re}} \qquad [10]$$

while, for the gas phase, a general expression is

$$f_G = C_G\left(\frac{D_G U_G \rho_G}{\mu_G}\right)^{-m}. \qquad [11]$$

In the case of laminar gas flow $C_G = 16$ and $m = 1$, whereas for a hydrodynamically smooth pipe at gas Reynolds numbers below 10^5, $C_G = 0.079$ and $m = 0.25$. Substitution of [7], [10] and [11] into [3] leads to a relation for the Lockhart-Martinelli (1949) parameter, X^2, defined as the ratio of the pressure gradient for liquid flow alone to the pressure gradient for gas flow alone and based on flowrates of gas and liquid existing for two-phase flow:

$$X^2 = \frac{-(\mathrm{d}P/\mathrm{d}x)_{SL}}{-(\mathrm{d}P/\mathrm{d}x)_{SG}} = \frac{\bar{U}_G^{(2-m)}}{\bar{U}_L^{n}} \cdot \frac{\bar{D}_L^{1+n}}{4\bar{D}_G^{m}} \cdot \frac{1}{\bar{A}_G}\left[\bar{S}_G + \frac{\pi}{4}\frac{\bar{S}_i}{\bar{A}_L} \cdot \frac{f_i}{f_G}\right]. \qquad [12]$$

If Taitel & Dukler's assumption that f_i is approximately equal to f_G is made, the equilibrium liquid level in the pipe, $h_L/D = \bar{h}_L$, is a function of X and n only. This results because all the dimensionless quantities on the r.h.s. of [12] depend on $\bar{h}_L$ only, according to the following relations derived by Taitel & Dukler:

$$\bar{A}_G = 0.25[\cos^{-1}(2\bar{h}_L - 1) - (2\bar{h}_L - 1)\sqrt{(1 - (2\bar{h}_L - 1)^2)}] \qquad [13]$$

$$\bar{A}_L = \pi/4 - \bar{A}_G \qquad [14]$$

$$\bar{A} = \bar{A}_L + \bar{A}_G \qquad [14a]$$

$$\bar{S}_G = \cos^{-1}(2\bar{h}_L - 1) \qquad [15]$$

$$\bar{S}_L = \pi - \bar{S}_G \qquad [16]$$

$$\bar{S}_i = \sqrt{[1 - (2\bar{h}_L - 1)^2]} \qquad [17]$$

$$\bar{U}_L = \bar{A}/\bar{A}_L \qquad [18]$$

$$\bar{U}_G = \bar{A}/\bar{A}_G. \qquad [19]$$

The liquid holdup, E_L, is given by

$$E_L = \frac{4}{\pi} \cdot \bar{A}_L. \qquad [20]$$

Thus

$$E_L = f(X, n). \qquad [21]$$

Beginning with [2], Taitel & Dukler derived a general expression for the Lockhart–Martinelli parameter, ϕ_G^2,

$$\phi_G^2 = \frac{-(\mathrm{d}P/\mathrm{d}x)_{tp}}{-(\mathrm{d}P/\mathrm{d}x)_{SG}} = \frac{\bar{U}_G^2 \cdot (\bar{D}_G \cdot \bar{U}_G)^{-m}}{4\bar{A}_G}\left(\bar{S}_G + \frac{f_i}{f_G} \cdot \bar{S}_i\right). \qquad [22]$$

If the assumption that $f_i \simeq f_G$ is made again, we may reduce this expression to

$$\phi_G^2 = \frac{\bar{U}_G^{2-m}}{\bar{D}_G^{1+m}} \qquad [23]$$

and since, by definition,

$$\phi_L^2 = -(\mathrm{d}P/\mathrm{d}x)_{tp}/-(\mathrm{d}P/\mathrm{d}x)_{SL} = \phi_G^2/X^2 \qquad [24]$$

then

$$\phi_L^2 = \frac{\bar{U}_G^{2-m}}{X^2 \cdot \bar{D}_G^{1+m}}. \qquad [25]$$

Thus, for drag reduction to occur in stratified flow,

$$\bar{U}_G^{2-m} < X^2 \cdot \bar{D}_G^{1+m}. \qquad [26]$$

If, for a particular flow the holdup, E_L, is known, this criterion may be used to determine whether drag reduction exists by evaluating [25] through [9], [12], [19] and [20] in conjunction with an appropriate value of m for turbulent gas flow. Substitution for X^2 in [25] using [12] gives:

$$\phi_L^2 = \frac{\bar{U}_L^{n}}{\bar{D}_L^{1+n}} \cdot \frac{(\bar{S}_G + \bar{S}_i)}{\left(\bar{S}_G + \frac{\pi}{4}\frac{\bar{S}_i}{\bar{A}_L}\right)}. \qquad [27]$$

This expression may be compared with that derived by Heywood & Richardson (1978), who used a simplified plug flow model for the intermittent flow regime in which the average gas and liquid velocities are assumed equal.

$$\phi_L^2 = \lambda_L^{1-n} = \bar{U}_L^{n-1} \qquad [28]$$

where λ_L is the input volume fraction of liquid defined as

$$\lambda_L = \frac{U_{LS}}{U_{LS} + U_{GS}}. \qquad [29]$$

In plug flow, it may be seen from [28] that the percentage drag reduction increases as n is reduced. In stratified flow, the effect of variation in n on $\phi_L{}^2$ may be determined by differentiation of [27]:

$$\frac{\partial \phi_L{}^2}{\partial n} = K' \cdot \log\left(\frac{\bar{U}_L}{\bar{D}_L}\right) \cdot \frac{\bar{U}_L{}^n}{\bar{D}_L{}^{1+n}} \qquad [30]$$

where

$$K' = \frac{\bar{S}_G + \bar{S}_i}{\bar{S}_G + \frac{\pi}{4} \cdot \frac{\bar{S}_i}{\bar{A}_L}} > 0. \qquad [31]$$

For a given $\bar{U}_L$, $\tilde{h}_L$ is constant and, therefore, so also is K'. Thus, $\partial\phi_L{}^2/\partial n$ is positive, indicating again that the percentage drag reduction increases as n is reduced, provided that $\bar{U}_L > \bar{D}_L$. By definition, $\bar{U}_L$ must always exceed unity, whereas $\bar{D}_L$ is less than unity for $E_L < 0.5$, and thus the criterion is fulfilled. In the range $0.5 < E_L < 1.0$, $\bar{D}_L$ exceeds unity and reaches a maximum value of about 1.22 at $E_L \sim 0.87$. By equating $\bar{U}_L$ and $\bar{D}_L$, the condition is defined where $\phi_L{}^2$ is independent of n. This is achieved by iterating for $\tilde{h}_L$ using [9], [13]–[16] and [18]. Values of important variables at this condition are

$$\bar{U}_L = 1.21; \qquad \bar{U}_G = 5.73; \qquad \bar{D}_L = 1.21$$

$$X = 11.3$$

$$\phi_L{}^2 = 0.752$$

$$E_L = 0.825.$$

These values hold for either laminar or turbulent gas flow.

An alternative approach to the prediction of drag reduction for a constant liquid flow rate is to calculate the increase in liquid flow rate when gas is injected into the pipeline under a constant pressure gradient. Asaturyan *et al.* (1973) presented their results for laminar-liquid, turbulent-gas stratified flow in a horizontal pipe in this manner. In the present model, the criterion of a constant pressure gradient is fulfilled by equating the two-phase pressure gradient in [1] with that created by the flow of liquid at a superficial velocity, U_{LO}. Thus,

$$\frac{1}{A_L}(\tau_{WL}S_L - \tau_i S_i) = \frac{2 f_{LO} U_{LO}^2 \rho_L}{D}. \qquad [32]$$

As before, the shear stresses, τ_{WL} and τ_i, are given by [4] and [5] while the friction factor, f_{LO}, may be expressed by a similar functional relationship as in [10]. The appropriate Reynolds number is

$$\mathrm{Re}_O = \frac{D^n U_{LO}^{2-n} \rho_L}{\gamma}. \qquad [33]$$

Substitution for τ_{WL}, τ_i and f_{LO}, and for A_L from [9] leads to

$$\frac{32\gamma U_{LO}^n}{D^{1+n}}\left[\left(\frac{U_L}{U_{LO}}\right)^n \cdot \left(\frac{D}{D_L}\right)^{1+n} - 1\right] = \frac{2 f_G \rho_G U_G{}^2 S_i}{D_L S_L}. \qquad [34]$$

The required ratio of the liquid flow rate in the presence of gas to that for liquid flow alone under a constant pressure gradient is given by

$$\frac{Q_L}{Q_{LO}} = \frac{U_{LS}}{U_{LO}} = \frac{A_L}{A} \cdot \frac{U_L}{U_{LO}}. \qquad [35]$$

The two-phase pressure gradient may be expressed from [2], [4], [5] and [9] by

$$-\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{tp} = \frac{2f_G U_G^2 \rho_G}{D_G} \qquad [36]$$

or, alternatively, by

$$-\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{tp} = -\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{SLO} = \frac{32\gamma U_{LO}^n}{D^{1+n}}. \qquad [37]$$

By combining [34]–[37] and introducing dimensionless quantities once again, we have

$$\frac{Q_L}{Q_{LO}} = \frac{\bar{S}_L}{\pi} \cdot \bar{D}_L^{(1+2n)/n} \cdot \left(1 + \frac{\bar{D}_G \bar{S}_i}{\bar{D}_L \bar{S}_L}\right)^{1/n}. \qquad [38]$$

Apart from n, all variables on the r.h.s. of [38] are functions of $\bar{h}_L$ only and are given by [9], [13], [14], [16] and [17]. Since liquid holdup is related to $\bar{A}_L$ by [20] and to $\bar{h}_L$ by [13] and [14],

$$\frac{Q_L}{Q_{LO}} = g(E_L, n). \qquad [39]$$

From a known available pressure gradient and known gas flow rate and density, $\bar{h}_L$ and hence E_L may be evaluated by iteration using [36]. The liquid flow rate ratio may then be obtained from [38]. Q_{LO} may be found by evaluating U_{LO} in [37] and hence the liquid flow rate in the presence of gas, Q_L, may be determined.

RESULTS OF COMPUTATION FROM ANALYSIS

Values of the Lockhart–Martinelli parameter ϕ_L^2 as predicted from [27] are presented in figure 2 for various n values corresponding to both pseudo-plastic and dilatant flow behaviour. Over the range $1 \leq \bar{U}_L \leq 1.21$, ϕ_L^2 decreases for a given $\bar{U}_L$ as n is increased from 0.1 to 2.0. However, the reverse is true for all $\bar{U}_L > 1.21$ and such behaviour is more in line with predictions of the plug flow model shown in figure 3, where, for all $\bar{U}_L$, ϕ_L^2 decreases as n is reduced for constant $\bar{U}_L$. Many instances of drag reduction ($\phi_L^2 < 1$) are evident in figure 2. The drag reduction effect occurs over the greatest range of $\bar{U}_L$ at the lowest n values, but, perhaps surprisingly, maximum drag reduction occurs at the highest n value plotted. Thus, a 36% maximum drag reduction occurs for $n = 2.0$ whereas a 25.5% value is predicted for $n = 0.1$.

In figure 4, ϕ_L^2 is presented as a function of the parameter X for various n values. Turbulent gas flow ($m = 0.25$) is assumed, and agreement is good between data for $n = 1.0$ and the corresponding predictions of Taitel & Dukler, who used an m value of 0.20 for turbulent gas flow. Over the plotted range $3.8 < X < 100$, the Newtonian liquid data ($n = 1.0$) depict $\phi_L^2 < 1$. However, the Lockhart–Martinelli correlation for laminar-liquid, turbulent-gas flow suggests that ϕ_L^2 never falls below unity at any X value in the range $0.01 < X < 100$. This is true also for their turbulent-liquid, turbulent-gas correlation. In general, for any power law model n value, the differences between predictions of laminar gas flow and those for turbulent gas flow were negligible. This suggests that the flow regime of the gas is of minor importance, although it will

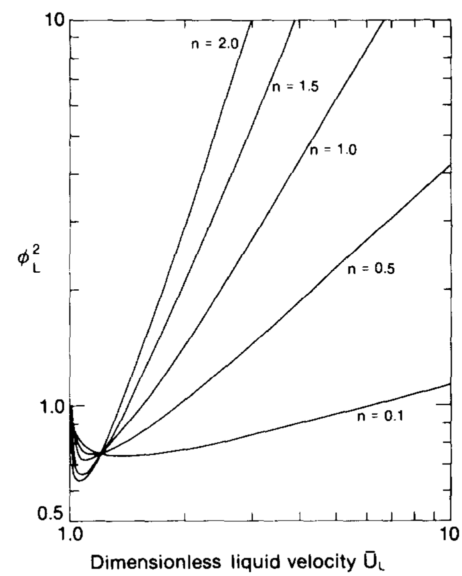

Figure 2. Lockhart–Martinelli parameter $\phi_L{}^2$ for stratified flow of gas and power law liquids.

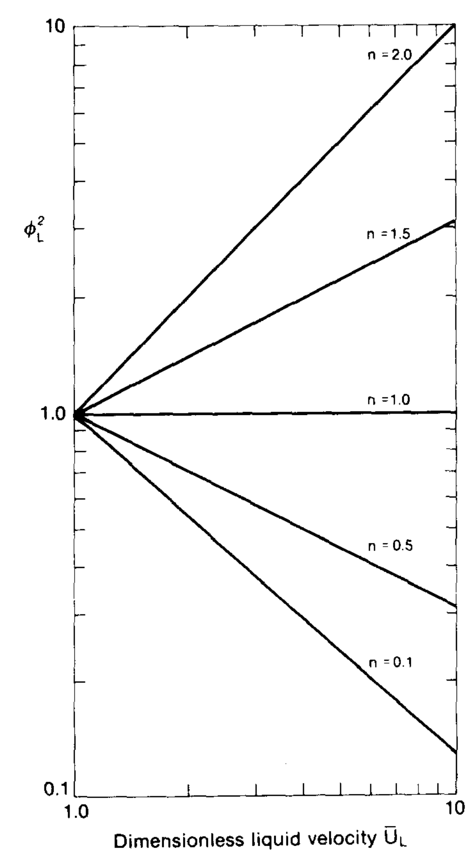

Figure 3. Lockhart–Martinelli parameter $\phi_L{}^2$ for idealised plug flow of gas and power law liquids

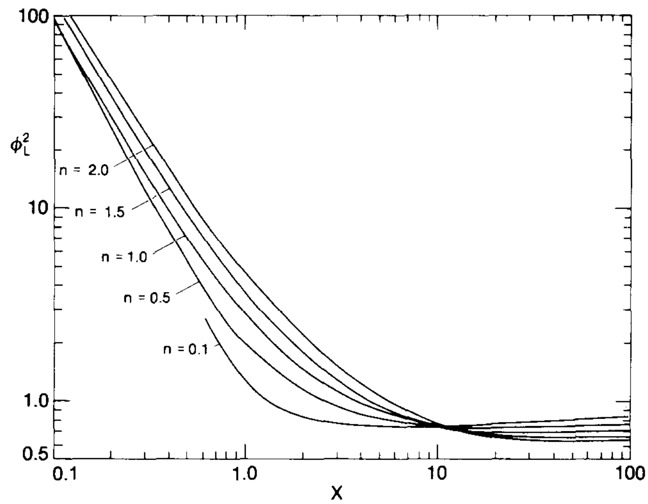

Figure 4. $\phi_L{}^2$ as a function of the Lockhart-Martinelli parameter X for stratified flow of gas and power law liquids (laminar liquid-turbulent gas).

undoubtedly be an influence on whether a smooth or wavy interface forms between the phases. In fact, Taitel & Dukler give a single curve for ϕ_G plotted against X, for either laminar or turbulent gas flow.

Maximum drag reductions are presented in figure 5 in the form of minimum values of $\phi_L{}^2$. As already noted, the maximum effect occurs at $n = 2.0$. An interesting suggestion from this curve is that maximum drag reduction is least for a liquid with $n \sim 0.25$, but increases once again for $n < 0.25$. The minimum value for $\phi_L{}^2$ of 0.714 obtained for a Newtonian liquid is consistent with previous findings for immiscible Newtonian liquid–liquid systems. For the case when both liquids are in laminar flow Charles & Redberger (1962) predicted by a numerical analysis of the Navier–Stokes equations that maximum drag reduction increases as the viscosity of the more viscous phase increases, until a constant value is reached at high viscosity of the more viscous phase. Using the specific case of an oil–water system, the minimum value of $\phi_M{}^2$ remained essentially constant at 0.763 for oil viscosities in excess of 0.1 Pa · s. Gemmel & Epstein (1962) predict a minimum of 0.709 for $\phi_M{}^2$.

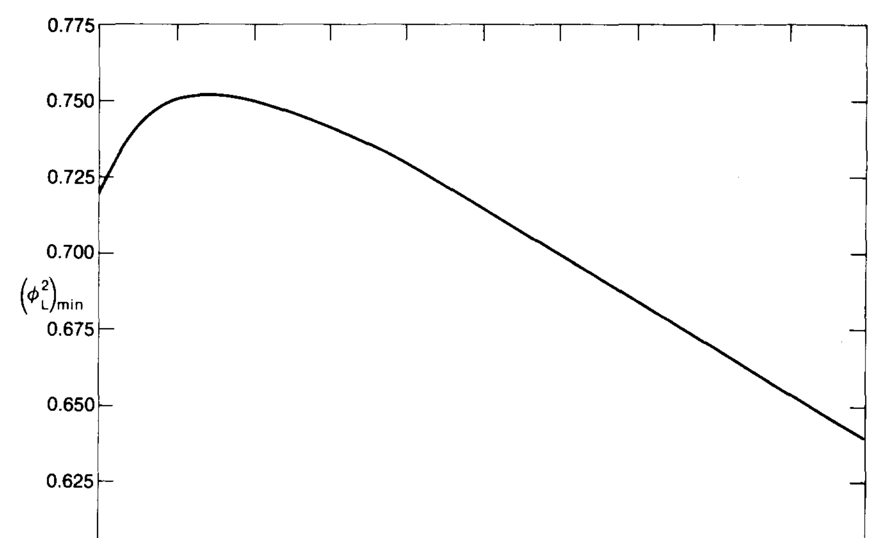

Figure 5. Maximum drag reduction obtainable for power law liquids.

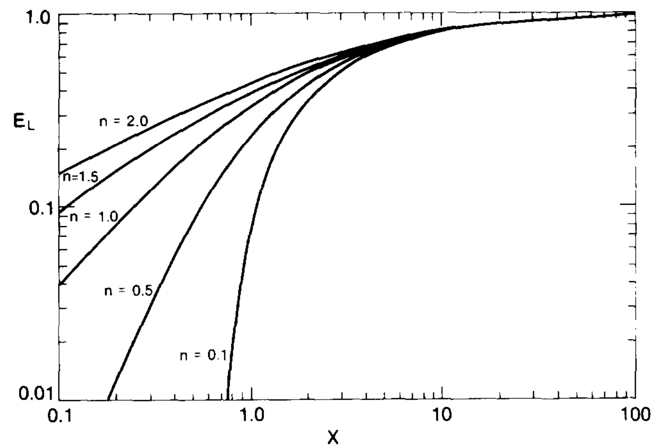

Figure 6. Average liquid holdup for power law liquids as a function of the Lockhart–Martinelli parameter X.

Liquid holdup is plotted in figure 6 as a function of X; the curves for various n apply equally well to either laminar-liquid/turbulent-gas or laminar-liquid/laminar-gas conditions. As a check, data for $n = 1.0$ agree with the Taitel–Dukler curve. It may be noted that for $X < \sim 7$, E_L decreases with decreasing n for a given X, whereas for $X > \sim 7$, E_L is independent of n. E_L^* values corresponding to $\phi_L{}^2 = 1$ are shown in figure 7. The plot may be interpretated by noting that any liquid holdup value which lies above the curve indicates drag reduction, whereas there is no drag reduction should holdup fall below the curve. Since, in stratified flow, pressure gradients are generally small and, therefore, difficult to measure accurately, the drag reduction condition may be more easily determined if n and E_L are readily measurable.

Predictions of the ratio Q_L/Q_{LO} for pseudo-plastic liquids are presented in figure 8. There exist large ranges of liquid holdup over which the ratio exceeds unity, indicating that liquid flow is increased by the presence of gas in these regions. The magnitude of the effect is greatest for highly pseudo-plastic liquids, the maximum values of the ratio occurring at low n values. The trend is continued for shear-thickening liquids in figure 9. However, it is interesting to note that even for a highly shear-thickening liquid with $n = 2.0$, the liquid flow may be increased by a maximum of 25% by injecting gas into the pipeline, but no advantage is to be gained if the liquid holdup is allowed to fall below ~ 0.73. Maximum values of Q_L/Q_{LO} for each n value studied are presented in figure 10.

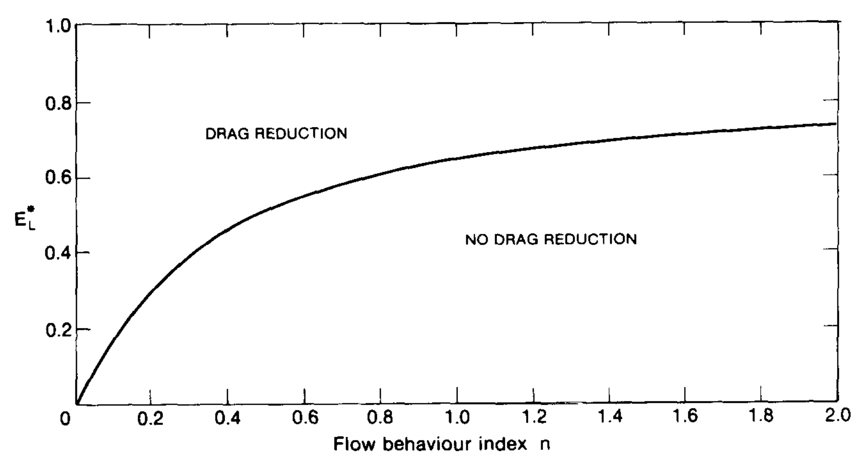

Figure 7. Critical values of average liquid holdup below which no drag reduction can occur in stratified flow.

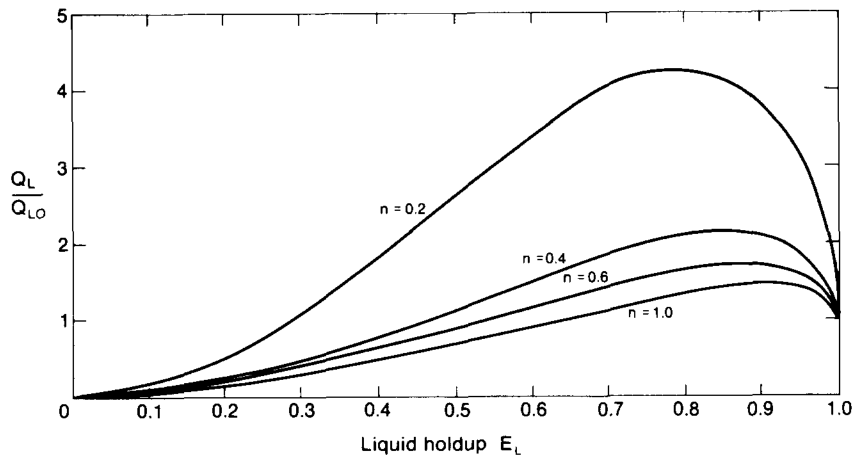

Figure 8. Increase in liquid volumetric flow rate by gas presence under constant average pressure gradient, for shear-thinning liquids.

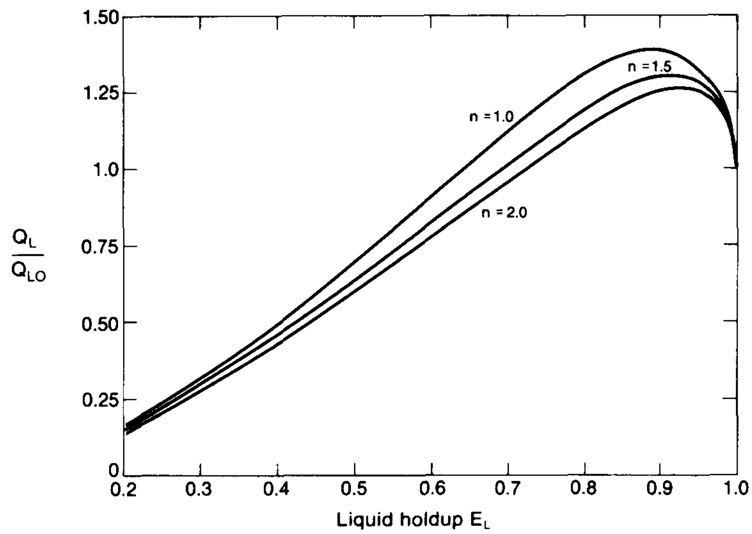

Figure 9. Increase in liquid volumetric flow rate by gas presence under constant average pressure gradient, for shear-thickening liquids.

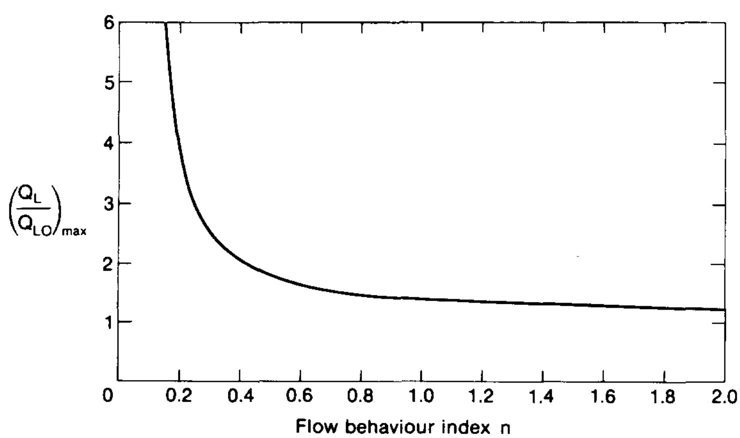

Figure 10. Maximum increase in liquid flow rate by gas presence under constant average pressure gradient.

CONCLUSIONS

By extending the two-phase, stratified flow analysis of Taitel & Dukler (1976) to include liquids having power law flow behaviour, pressure gradient predictions may be presented by the Lockhart–Martinelli (1949) parameter ϕ_L^2 as a function of X and the flow behaviour index, n. It has been found that liquid holdup is also a function of X and n only. Over the range $0.1 \le n \le 2$, conditions are predicted where the two-phase pressure gradient is less than the pressure gradient for liquid flow alone at the same liquid volumetric flow rate ($\phi_L^2 < 1$) provided the liquid flow is laminar. This drag reduction is likely to occur over the largest range of liquid holdup at low n values, but the maximum effect is predicted to take place for liquids with $n \geq 2$. Such behaviour is somewhat surprising since a simple plug flow model for intermittent flow applied by Heywood & Richardson (1978) to power law suspensions suggests that drag reduction will not occur at all for shear-thickening materials.

The ratio of the liquid flow rate in the presence of gas to that existing for liquid flow alone, under a constant pressure gradient in both cases, increases for a given liquid holdup as n is reduced in the range $0.2 \le n \le 2.0$. The range of E_L over which the ratio is greater than unity also increases as n is reduced. Maximum values of the ratio rise rapidly at low n values, indicating that the presence of gas is most advantageous in increasing liquid flow for highly shear-thinning liquids.

Although no known experimental data are available at present to verify predicted behaviour of gas/non-Newtonian systems, the Taitel & Dukler approach appears basically sound in view of its ability to describe well pressure drop data for both gas/Newtonian liquid and Newtonian liquid/liquid stratified flows. A series of fundamental experiments is now required to test current predictions for gas/non-Newtonian systems.

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